

8)

$$a_n = 2a_{n-1} + a_{n-2}$$

$$a_n - 2a_{n-1} - a_{n-2} = 0$$

Sea $dr^n = a_n$

$$dr^n - 2dr^{n-1} - dr^{n-2} = 0$$

$$dr^n - 2(r^{n-2} - 2r - 1) = 0 \quad \text{E.C}$$

$$r^2 - 2r - 1 = 0$$

$$R_1 = 1 + \sqrt{2}, R_2 = 1 - \sqrt{2} \quad \text{R.C}$$

$$a_n = C_1 R_1^n + C_2 R_2^n$$

$$a_n = C_1 (1 + \sqrt{2})^n + C_2 (1 - \sqrt{2})^n$$

$$a_0 = C_1 (1 + \sqrt{2})^0 + C_2 (1 - \sqrt{2})^0$$

$$1 = C_1 + C_2$$

$$a_1 = C_1 (1 + \sqrt{2})^1 + C_2 (1 - \sqrt{2})^1$$

$$2 = C_1 (1 + \sqrt{2})^1 + C_2 (1 - \sqrt{2})^1$$

$$2 = 1 + \sqrt{2} C_1 + 1 - \sqrt{2} C_2$$

$$C_1 + C_2 = 1$$

$$1 + \sqrt{2} C_1 + 1 - \sqrt{2} C_2 = 2$$

Es un sistema de ecuaciones

$$C_1 = \frac{2 + \sqrt{2}}{4}$$

$$C_2 = \frac{1 - \sqrt{2}}{-2\sqrt{2}}$$

Por lo tanto, la solución general es:

$$a_n = \frac{2 + \sqrt{2}}{4} (1 + \sqrt{2})^n + \frac{1 - \sqrt{2}}{-2\sqrt{2}} (1 - \sqrt{2})^n$$