Introduction to Robotics (CS223A) Homework #3

(Winter 2007/2008) Due: **Wednesday, February 06**

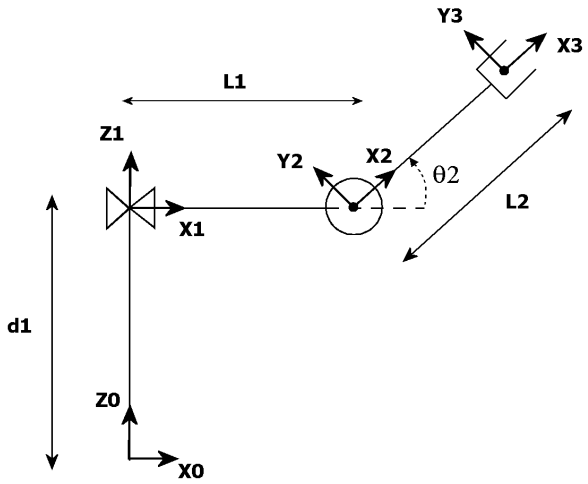
1. You are given that a certain RPR manipulator has the following transformation matrices, where $\{E\}$ is the frame of the end effector.

$$ {}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; {}^0_3T = \begin{bmatrix} c_1c_3 & -c_1s_3 & -s_1 & L_1c_1 - s_1d_2 \\ s_1c_3 & -s_1s_3 & c_1 & L_1s_1 + c_1d_2 \\ -s_3 & -c_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^0_ET = \begin{bmatrix} -s_1 & c_1s_3 & c_1c_3 & L_1c_1 + L_2c_1c_3 - s_1d_2 \\ c_1 & s_1s_3 & s_1c_3 & L_1s_1 + L_2s_1c_3 + c_1d_2 \\ 0 & c_3 & -s_3 & -L_2s_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

   Derive the basic Jacobian relating joint velocities to the end-effector's linear and angular velocities in frame $\{0\}$.

2. Consider the planar PR manipulator shown here:

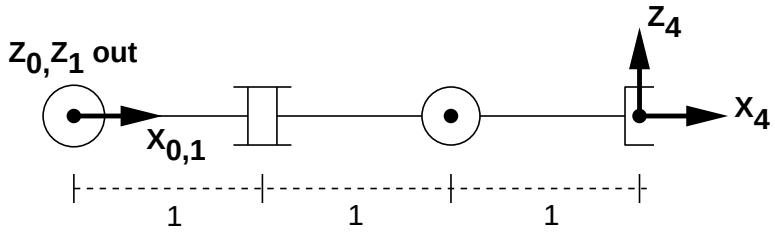


   (a) Find the origin of frame $\{3\}$ expressed in terms of frame $\{0\}$, that is ${}^0\mathbf{P}_{3org}$.
   Tip: you can derive this geometrically, if you want to avoid going through DH parameters.

   (b) Give the 2×2 Jacobian that relates the joint velocities to the linear velocity of ${}^0\mathbf{P}_{3org}$.

   (c) For what joint values is the manipulator at a singularity? What motion is restricted at this singularity?

3. Consider the RRR manipulator shown here:



Note: in the figure, the numbers below the links represent the lengths.

(a) Find the DH parameters for this manipulator. Remember to assign the interior frames of this manipulator using the conventions discussed in class.

i	α_{i-1}	a_{i-1}	θ_i	d_i
1				
2				
3				

(b) Derive the forward kinematics, ${}^0_4T$, of this manipulator.

(c) Find the basic Jacobian, J_0, for this manipulator.

(d) Find ${}^1J_v$, the position Jacobian matrix expressed in frame $\{1\}$.

(e) Use the matrix that you found in part (d) to find the singularities (with respect to linear velocity) of this manipulator.

(f) For each type of singularity that you found in part (e), explain the physical interpretation of the singularity, by sketching the arm in a singular configuration and describing the resulting limitation on its movement.