Introduction to Robotics (CS223A) Homework # 1

(Winter 2007/2008) Due: **Wednesday, January 23**

Some tips for doing CS223A problem sets:

- Use abbreviations for trigonometric functions (e.g. $c\theta$ for $\cos(\theta)$, s_1 or $s\theta_1$ for $\sin(\theta_1)$) in situations where it would be tedious to repeatedly write sin, cos, etc.
- Unless instructed otherwise, leave square roots in symbolic form rather than writing out their decimal values.
- If you give a vector as an answer, make sure that you specify what frame it is given in (if it is not clear from context). The same rule applies to rotation and transformation matrices.

1. A frame $\{B\}$ and a frame $\{A\}$ are initially coincident. Frame $\{B\}$ is rotated about $\hat{Y}_B$ by an angle θ, and then rotated about the new $\hat{Z}_B$ by an angle ϕ. Determine the 3×3 rotation matrix, ${}^A_BR$, which will transform the coordinates of a position vector from ${}^B\mathbf{P}$, its value in frame $\{B\}$, into ${}^A\mathbf{P}$, its value in frame $\{A\}$.

2. We are given a single frame $\{A\}$ and a position vector ${}^A\mathbf{P}$ described in this frame. We then transform ${}^A\mathbf{P}$ by first rotating it about $\hat{Z}_A$ by an angle ϕ, then rotating about $\hat{Y}_A$ by an angle θ. Determine the 3×3 rotation matrix operator, $R(\phi, \theta)$, which describes this transformation.

3. (a) Given a transformation matrix:

$$ {}^B_AT = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \cos(\theta) & -\sin(\theta) & 2 \\ 0 & \sin(\theta) & \cos(\theta) & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

   Find ${}^A_BT$

   (b) Given $\theta = 45^\circ$ and ${}^BP = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}^T$, compute ${}^AP$.

4. Given the following 3×3 matrix:

$$ R = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix} $$

   (a) Show that it is a rotation matrix.

   (b) Determine a unit vector that defines the axis of rotation and the angle (in degrees) of rotation.

   (c) What are the Euler parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ of R?